

Suppose that in the country of Knights and Knaves you meet three individuals, A, B, and C. You discover that at least one of them is a Knight and at least one of them is a Knave.

A says "B or C is a Knight" and B says "A or C is a Knight."

Which of them are Knights and which are Knaves?

#### THE BOOLEAN CONNECTIVES Monday, 27 January

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1.4.

Monday, January 27, 2014

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Note: SameSize(¬a, b) is NOT a sentence.

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SameSize(a, b) ∨ Cube(a)

 $\neg$ Cube(a)  $\land$  a=b

 $\neg$ (Cube(a)  $\land$  a=b)

 $\neg$ (Cube(a)  $\land$  a=b)  $\lor$  (Larger(b, c)  $\lor \neg$ Medium(b))

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  (A ∨ B) ∧ C is a conjunction the ∧ is the m.c.
  (A ∨ B) and C are the conjuncts
  Parentheses are used to determine the order of the
  - connectives and disambiguate sentences.

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#### $\neg(A \lor (B \land \neg C)) \land (\neg(D \land E) \lor F)$

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 $\neg(A \lor (B \land \neg C))$ 

 $(\neg (D \land E) \lor F)$ 

F

 $(A \lor (B \land \neg C)) \qquad \neg (D \land E)$ 

 $\neg(A \lor (B \land \neg C)) \land (\neg(D \land E) \lor F)$ 

¬(A ∨ (B ∧ ¬C))

 $(\neg (D \land E) \lor F)$ 

F

(A ∨ (B ∧ ¬C))

(B ∧ ¬C)

¬(D ∧ E) | (D ∧ E)

Α





 $(\neg (D \land E) \lor F)$ 

F

(A ∨ (B ∧ ¬C))

A (B ∧ ¬C) B ¬C  $\neg(D \land E)$  $(D \land E)$ D E





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A	٦A
TRUE	FALSE
FALSE	TRUE

- Low And Black of the Street St.

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 We translate A A B into English as 'it is the case that A and it is the case that B'.

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• The sentence A  $\wedge$  B is true iff A and B are both true.

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Α	В	A ∧ B
TRUE	TRUE	TRUE
TRUE	FALSE	FALSE
FALSE	TRUE	FALSE
FALSE	FALSE	FALSE

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- In English, 'conjunction' refers to a part of speech, such as 'but', 'or', 'yet.'
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- 'And' in English can link two names or properties.
   For example, Sam and Sarah had breakfast; Sam had breakfast and went to the park.
   In FOL, conjunction only links two sentences.
- In English, 'and' is often used to imply causation or a temporal sequence.

'And' does not have this implication in FOL.

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- Truth table for disjunction:

Α	В	$A \lor B$
TRUE	TRUE	TRUE
TRUE	FALSE	TRUE
FALSE	TRUE	TRUE
FALSE	FALSE	FALSE

 In English, 'A or B' is often used to mean that either A is true, or B is true, but not both (exclusive or).

In FOL, exclusive or (exactly one of) could be expressed as  $(A \lor B) \land \neg(A \land B)$ 

The simple disjunction  $(A \lor B)$  is always the inclusive or (at least one of and maybe both).

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Monday, January 27, 2014

#### Alice is at the party and Bill is at the party

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Alice is at the party and Bill is at the party  $P(a) \land P(b)$ 

Contractions and all

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