Why Indispensability is Not an Argument for Mathematical Realism

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There are very few arguments that have been advanced for mathematical realism, that is, the position that mathematical objects exist. In fact, if you believe Hartry Field, there is only one non-question begging argument that has even been put forward; this is the indispensability argument. Whether or not this is the only available argument to the realist, it certainly is one of the most often used and is worthy of serious consideration.

Although I don't think that there is one clear mistake in the indispensabilist's argument, I will argue that if you hold what is a very standard view, the view that mathematical objects are abstract, then there is no reason to accept the indispensability argument as a justification to believe that mathematical objects exist.

What is the argument claiming?

The indispensability argument attempts to show that we have justification for believing that mathematical objects exist. An early version of the argument was given by Hilary Putnam, who referred to Quine's views on some of these matters, and thus it is often called the Quine-Putnam indispensability argument, though the inference from the applicability of mathematics to its truth and thereby to realism goes back at least to Frege. More recently, the argument has been spelled out in a more formal way by several different authors, the main version being something like:

1. We ought to have ontological commitment to all and only those entities that are indispensable to our best scientific theories.
2. Mathematical objects are indispensable to our best scientific theories.

   Therefore,

3. We ought to have ontological commitment to mathematical entities.

This particular formulation is due to Mark Colyvan who argues for its soundness in his recent book *The Indispensability of Mathematics*. (Colyvan, 2001, pg.11)

It is clear that the argument as stated is valid. Thus the only remaining question is whether the premises are in fact true. But what exactly are these premises claiming? Since both of the premises focus on our best scientific theories, I think a reasonable place to begin examining the argument is with this phrase.

Our best scientific theories

At first, it may seem obvious that our best scientific theories are the ones that good scientists today use to make predictions. However, if that were the case, I don't think that the argument would be as strong as we might want it to be. In fact, premise one would almost certainly be false. As Penelope Maddy points out in "Indispensability and Practice", the actual attitudes of working scientists vary "from belief to grudging tolerance to outright rejection." (Maddy, 1992, p. 280). Part of her objection stems from scientists' use of instrumental portions of theory to aid in their calculations. Scientists talk of frictionless planes, infinitely deep water, and incompressible fluids among many other things. It seems clear that just because scientists use them in their theories, that doesn't imply that we ought to be committed to their existence.

Even worse than this is the fact that in many cases the theories that scientists use today are inconsistent with each other at points, as is the case with Quantum Mechanics and General Relativity. Each theory makes extremely precise predictions in the realms in which they are used; yet they imply different things about the nature of the universe and the objects within it.

Surely we should not have ontological commitment to all that both theories entail since part of at least one of them must be false.

So if there is something to this argument, it must be that what we are referring to in our premises is not the theories that scientists use on a daily basis. If we think of the case of the frictionless planes, it seems clear that while scientists may use them for calculations in some cases, this use seems to be purely pragmatic. The scientists in question don't really think that the theory they are using is true.

In most of these cases, it is not merely that these idealizations are time-savers and thus are pragmatic in that sense, but that we don't actually know what the correct theory is. We couldn't simply plug in the numbers to some complicated equation even if we wanted to, for we don't know the equation. While this is true, scientists still don't believe that these idealized objects exist since they believe that in principle, we have access to a more accurate theory of what is actually going on. I say "in principle" because no scientist today could even dream of using quantum mechanics to predict what is going to happen when we allow water to flow through a pipe. However, the attitude is that whether or not we could ever know enough to be able to apply theories that operate on smaller and smaller scales until we could predict the phenomena even more accurately that our best theories today using the idealizations, we do know that there are such theories. Thus the idealizations are not part of the "best" scientific theories.

The case is similar with inconsistent theories. Although each is well confirmed now, we think that as we learn more, our best theories will not be inconsistent with each other. If not, we would have proof that our theories were false. In any case, it seems that whatever our "best scientific theories" refers to, it is not to the theories that scientists actually use. What it refers to is our best guess as to what our ultimate scientific theories will be. Since right or wrong, we have the attitude that science progresses in a way that we can somehow expect, while we may not have an actual theory that has been confirmed by examining water flowing through pipes, we expect that we will at some time in the future, and that incompressible fluids will not be part of this theory.

Indispensable

The Indispensability argument as stated above is what its defenders are actually trying to defend. What premise two of this argument claims is that mathematical objects are indispensable to these future scientific theories. "Indispensable" is another notion that needs serious clarification. What Putnam actually claims in his initial statement of the view in "Philosophy of Logic" is that quantification over mathematical entities is indispensable to science. Since our best scientific theories would say things of the sort "There exists a function such that …", we should believe that functions exist.

What Putnam may have in mind is something as simple as "scientists do actually quantify over numbers when doing science." We have seen that we should alter this to what we think our best science might be in the future. But the fact that our future scientists will use mathematics in doing science does not by itself imply that mathematics is indispensable to it. According to Colyvan, what is means for an entity to be dispensable to a theory is that:

> 1) There exists a modification to the theory in question resulting in a second theory with exactly the same observational consequences as the first, in which the entity in question is neither mentioned nor predicted.

2) The second theory must be preferable to the first.

One interesting point to note is that Colyvan actually thinks that this new theory must be superior and not just “at least as good.” It seems to me though, that if there were such a theory as this, then our first theory would not have been our best scientific theory in the first place. If this is what Colyvan has in mind for the argument, he might simply say, “is part of our best scientific theories’ rather than bothering with the possibly confusing reference to its “indispensability.” However, Colyvan may run into a problem if his version of the argument is formulated this way. Imagine two scientific theories, one a modification of the other, which have the same empirical consequences. Perhaps one theory allows for electrons traveling backward through time in order to account for certain phenomena while the other employs forward time moving positrons. Isn’t it possible that these theories are simply equally as good? If so, should we be committed to believing in particles that travel backward in time or to antimatter particles? These theories are inconsistent, but on a purely empirical level, there might seem to be no deciding between them, at least at this point in time.

Colyvan gives what he considers to be reasons for thinking that one theory is superior to another (assuming that they are empirically equivalent.) He considers simplicity, explanatory power, fruitfulness, and elegance all as criteria for judging between theories. It certainly seems possible that our two theories fare equally well in all these categories. Or perhaps one is more elegant but the other requires fewer entities. How are we to judge between the two? What is to be considered part of our best scientific theories? If we picked both as part of our best theories, then we would be committing ourselves to belief in both the existence of antimatter particles and to electrons that travel backward in time. But this certainly seems to be the wrong answer. Why be committed to both when simplicity demands that we only need one of these entities for our theory? Imagine that, for whatever reason, we pick the antimatter theory (as we currently do). The anti-matter particles become indispensable because by getting rid of them our theory would be made worse. But I would argue that this seems to be the wrong conclusion. We shouldn't claim that antimatter particles are *indispensable* to our best scientific theories since we could do perfectly well without them if we had some reason we wanted to get rid of them. I think here the right conclusion is that each entity is dispensable.

Hartry Field, in *Science Without Numbers*, attempts to show that mathematics is dispensable to science. While he doesn't formally discuss exactly what this would take, he does think that it must be the case that if we could get rid of all of the references to mathematical objects in our theory and the remaining theory was still empirically adequate and attractive in a certain way, then mathematics is dispensable to that theory. While this may leave us in the dark as to whether or not some particular nominalization of a theory is actually still attractive, I think that Field, rather than Colyvan, has the right idea here. It isn't required that getting rid of mathematics makes the theory better. It should be merely required that it is possible to get rid of mathematics and that the remaining theory would be still be acceptable in a certain way. In this case, getting rid of antimatter particles in favor of backward time-traveling particles might not be superior, but it is possible and would still be an acceptable scientific theory. Thus antimatter particles are dispensable.

Although this could make the argument convincing to fewer people (mathematics may be dispensable in Field's view but not Colyvan's), it strengthens the plausibility of the first premise and I think enhances the power of the argument over all. I tend to think that it also gives us a notion of indispensability that is closer to what we mean when we state the argument in the first place. We literally mean something like "You can't do science without math." We don't mean, "If you did science without math, your science would at best be equally as good, but it wouldn't be better."

Exactly which mathematical objects are indispensable?

Although the orthodox mathematical realist will believe that the justifications of various types of mathematical objects are all on par, this is not going to be the case if our justification stems solely from indispensability. One might ask, "Exactly which mathematical objects are indispensable?" A naïve view is that if anything, surely the natural numbers are used most often in science and so are indispensable. However, if we hold to the view of indispensability above, one does not need to quantify over natural numbers in our scientific theories. We can get by with quantifying over some larger class and then restricting our sentences. Or we could use some substitute, such as quantifying over sets. At various points in his writings, Quine himself thought that sets were the only mathematical objects needed for scientific theories and thus the only mathematical objects we have reason to believe in. But we can find various substitutes for sets, such as structures of certain sorts in category theory. Or perhaps we can go back to using ordinary real numbers for science. At best, it is unclear what is going to be needed, but it would seem that no particular mathematical objects are necessarily quantified over. There is always going to be an equivalent theories quantifying over different mathematical objects that could serve us equally well. Thus the indispensability argument can't really be used by itself to justify belief in any particular mathematical objects.[1] Typically, it would be combined with some sort of coherence view or constructivist view allowing us to add more objects once we have some. Here I will not deal with these, I will simply treat the indispensability argument as initial justification that some mathematical objects exist without reference to which particular objects they are. Thus it is an argument for mathematical realism where realism is that some mathematical entities exist, not an argument for the existence of any particular entities.

Who should believe it?

So at this point what the argument seems to be stating is that: 1) In our best estimate, our best scientific theories in the future will quantify over some mathematical objects and 2) There is no possible scientific theory that is empirically equivalent that doesn't do this quantification. Since both of these are the case, we should believe that some mathematical objects do exist.

Now the question becomes, why should we believe these claims? This view that we should be ontologically committed to what we quantify over comes from Quine who says "Ordinary interpreted scientific discourse is as irredeemably committed to abstract objects – to nations, species, numbers, functions, sets – as to apples and other bodies. All these things figure as values of the variables in our overall system of the world. The numbers and functions figure just as genuinely in physical theory as hypothetical particles." (Quine 1981 pg. 149) Here it is clear that Quine's basic argument is that since scientific discourse makes reference to hypothetical particles and therefore we think they exist, when we make reference to abstract objects we should treat them the same way.

There are at least three important things going on in the passage from Quine. He assumes that we should interpret all scientific discourse in an ordinary fashion, that we have reason to believe in hypothetical particles, and that science treats mathematical objects in the same way as physical objects. All three of these views could be, and have been, questioned.

You might think that mathematical existence claims should not be taken at face value claiming instead that when we say "There exists a prime number greater than five we mean something like "In the story of mathematics, there is a prime number greater than five." Another possibility is that you think that to claim, "There is a set of all prime numbers less than 100" is to claim that "We can construct a set of all the prime numbers less than 100." Perhaps you think mathematical existence claims are merely consistency claims. In any of these views, or others where mathematical existence claims are not taken at face value, there is no reason to accept Quine's statement that we should accept as existing what we quantify over and thus no reason to accept the indispensability argument.

Quine's version of the argument for mathematical realism also clearly depends on a view about scientific realism. Someone like Bas van Fraassen who argues that scientific theories need only to be empirically adequate to be our best scientific theories would balk at being obligated to accept hypothetical particles into his ontology; so the analogy to mathematical objects carries no weight. The fact that we should treat mathematical object in the same way we treat physical objects lest we be hypocritical does not lead to the conclusion that we should believe that mathematical objects exist unless you already believe that all of the physical objects quantified over in this way also exist.

Lastly, not only do you need to take mathematical existence claims at face value, and be willing to admit into our ontology all physical objects quantified over, but you must also be willing to treat mathematical objects in the same way as physical objects. Quine's view on this matter is that the mathematical claims and physical claims are all part of the same theory as a whole and thus get tested, confirmed, or disconfirmed as a whole. This view is called confirmational holism. If you held a different view, for example the view of Elliott Sober, you might think that you only confirm scientific hypotheses when you test them against other hypotheses. The key claim here is that you don't confirm or disconfirm what is common to all of the hypotheses being tested – in this case, the background mathematics. Thus the mathematics is not being confirmed just because the hypothesis is being confirmed.

While van Fraassen thinks we should separate observables from the unobservables and treat them differently, Sober believes that we should separate the hypothesis being tested from the auxiliaries being used, that is, the assumptions made to carry out the test. These are never confirmed or disconfirmed along with the hypothesis and thus are treated differently from scientific hypotheses.

So you could fail to be persuaded by the indispensability argument if you don't take mathematical existence claims at face value, if you aren't a scientific realist, or if you don't think mathematical claims should be treated the same as other scientific claims. But what if you accept all that? This is in fact what Resnik, Colyvan, and others take as all that is needed to accept the indispensability argument. However, I would argue that even granting all that, there are still many reasons to deny that the argument is sound.

Premise one

The first premise of the argument claims that we ought to have ontological commitment to "all and only" those entities that are indispensable to our best scientific theories. Of course Colyvan realizes that he only needs the 'all' in order for the argument to be valid, but he puts the 'only' in as well in order to "highlight the important role that naturalism plays in questions about ontology, since it is naturalism that counsels us to look to science and nowhere else for answers to ontological questions." (pg. 12)

However, there are significant dangers in this approach. As is obvious, the change now makes the first premise much stronger and therefore, much more likely to be false. A defender of the argument might simply point out that anyone who agrees with the ‘all’ clause of the argument but not the ‘only’ half should simply ignore it as unnecessary. This is very dangerous as psychologically, the reader may link this ‘full-blown naturalism’ of Colyvan’s with the indispensability argument and thus dismiss it out of hand without considering the possibility of weakening the premise and examining the argument further.

Before adjusting the premise, it is worth asking whether the change will actually affect anything. For example, if the only reason to believe the weaker version automatically provides a reason to believe the stronger version (as Colyvan seems to think) then we might as well leave in the full version since it won’t really affect anything. I believe that this is not the case. It is true that in the philosophical community, many philosophers recognize science as a source of knowledge about ontological claims. How exactly this gets cashed out will affect whether or not the reader is likely to accept the first premise. However, far fewer philosophers accept the relatively extreme naturalism of Colyvan in accepting the view that science is the only possible source of knowledge about ontological claims. At first blush, this would seem to rule out many forms of apriori knowledge that would have many ethicists and metaphysicians up in arms. Thus it is easy to see that there are many philosophers that would accept the weakened version of premise one but not the stronger version. Since the full-blown naturalism of Colyvan is not essential to mathematical realism, we will consider here only the weaker version of premise one.

Contingent, independently existing mathematical objects

Imagine that you have the view of mathematical objects that Colyvan and Resnik have: mathematical objects may or may not exist and that science gives us a reason to think that they do. Since the indispensability argument provides us with empirical evidence for the existence of these objects, it seems as though it may be possible to have scientific evidence against their existence. Some views of confirmation even require this to be the case. However, if it is in fact possible to have evidence against their existence, it would seem that their existence must be a contingent matter since it would be hard to imagine having evidence that a necessarily existing object does not exist.

As Colyvan point out, although this may be the natural position for the indispensabilist to take, it may not be the only consistent view to hold. As Kripke argued, it may be possible that there are necessary truths that we can have only aposteriori knowledge of. [2] Perhaps statements of mathematical existence are like this. I will address this view later in the paper. For now, I will address only the view that the existence of mathematical objects is contingent.

Since the current view being addressed holds that it is possible that mathematical objects do not exist, it makes perfect sense to ask the question, “What would the world be like if they did not exist?” The traditional view of mathematical objects would hold that their existence is not dependent on the existence of anything else and that nothing else that exists depends on them. They may be counterexamples to this extremely simplistic picture such as the set containing me as its only member (which depends on me for its existence) but it seems clear what is intended here – that the number 2 and similarly ‘pure’ mathematical objects do not depend on any physical objects for their existence.

If this were the case, the physical world might be, for all we could tell, exactly the same. But what of our best scientific theories? We can't assert that the theories would remain the same otherwise the indispensability argument could still be made, and we would have justification for believing that mathematical objects do exist even though they didn't. Furthermore, this evidence would be the same evidence that we have now, thus it would not really be evidence is any correct sense of the world. Anyone who claims science gives us evidence for mathematical realism must also believe that this evidence would be absent if realism were false. Thus it is clear that the indispensabilist must claim that our best scientific theories would actually be different if in fact mathematical objects did not exist.

But how could this be so? After all, the indispensabilist is a scientific realist and thus believes that what science is trying to do is to describe the happenings of the physical world. But would the physical world be somehow different if mathematical objects did not exist? The answer must be yes for the argument to carry any force. If we assume that mathematical objects are abstract, then it would seem they would have to be acausal. Maddy once held that this was not necessarily the case (see Maddy 1990) but she has since changed her view arguing that her previous view was untenable (see Maddy 1997). If mathematical objects are acausal, it would seem that there can't be any actual activity in the physical world that would be any different, therefore it must be the case that we would merely describe what does happen differently. The question is then why it would be necessary for science to describe the world differently even though the physical world was exactly the same? Of course we should not be overly hasty here and assume that the only factors that can make a difference are causal factors, but causal power is the natural model to think of when thinking of why the world would be different if some physical object didn't exist (say if I did not exist). The burden is on the realist to explain how mathematical objects can make a difference to physical theory if causation is not the model they have in mind.

Similar to this criticism are criticisms by Cheyne and Pigden that claim that nothing could be indispensable to science unless it was causally active. Colyvan defends his view against this criticism by arguing that built into this criticism is the assumption that all explanations are causal. He then defends this indispensability doctrine by arguing against this view of explanation. It would be difficult for Colyvan to avoid charges that he is simply knocking down a straw-man argument here. He is clearly taking for granted that the only reason that something would be indispensable to scientific theories is that it has explanatory value, otherwise his defense would have no force. Even if he is right about this point, the particular view of explanation that he defends in order to push home his view that mathematical objects are explanatorily relevant fails.

The notion of explanation that Colyvan is working with is extremely broad here, and purposely so. He claims that all he is assuming is that "an explanation must be enlightening – it must make the phenomena being explained less mysterious." He then goes on to give what appear to be several mathematical explanations for certain physical phenomena. Since mathematical objects are not causal, if these are genuine explanations, then not all explanations are causal. However, the examples he gives don't convince me that the *ontological status* of mathematical entities is playing any part in the explanation.

The ideal example that he gives for this context is the explanation of why it is that at any given time there are two points on the surface of the earth that have exactly the same temperature and pressure. This seems to be a nominalistically stateable sentence. The explanation that he offers is a mathematical proof showing that if the earth is homologous to a sphere and temperature and pressure are continuous functions across its surface, then at any given time *t* there will be two antipodal points that have the same temperature and pressure.

While this explanation appears to make reference to continuous functions and spheres, this explanation does not require that these abstract entities exist. This explanation could be seen as taking a certain deductivist form. If the earth had certain properties, then treating it like a sphere would not lead us to say anything false about it, and if temperature and pressure varied continuously… then the earth would have certain physical properties. This explanation doesn't make any conditional existence claims about functions or spheres whatever. In other words, our explanation would still be a good one, even if no abstract entities of these sorts existed at all. The example is intended by Colyvan to show not only that some explanations are acausal, but is also intended to show us how mathematics can fit in to our scientific theories of the world. However, it is certainly not clear from this explanation that a fictionalist story couldn't explain the phenomena equally well.

What I have been suggesting here is that if you believe that mathematical objects exist independently of all physical objects in a strong sense of not interacting with the physical world, then our scientific theories would be exactly the same whether or not mathematical entities actually existed. If this were true, it is hard to see how our theories could be providing evidence that they do in fact exist, thus undermining the indispensabilist's claims.

Contingently existing non-independent objects

If we believe that the existence of mathematical objects could not change the world in any direct way, the indispensabilist is forced to argue that the physical world and its makeup can actually determine which mathematical objects exist so that the dependence relation actually runs in the physical to the abstract direction. A view like this might be that mathematical objects exist because physical objects instantiate certain relations to other physical objects. In the case of antipodal weather points, you could imagine someone believing that these continuous functions do exist because temperature and pressure vary in the right sort of way. Thus we have evidence of mathematical realism in virtue of having evidence that physical objects fall in certain relations to each other.

Certain modal structuralist views are very much like this. The claim might be something like this: Mathematical objects are just parts of certain structures. These structures exist contingently; they exist if and only if there are some physical objects which instantiate the properties of the given structure. Since objects in the physical world have certain properties, certain mathematical objects do exist.

One problem of this view is how to justify objects that no physical objects actually instantiate. One might think that if one real-to-real function exists, then they all do. But this would seem impossible since there are just not enough physical objects for this to be true. This view runs the danger of having a largest existing number or having some real numbers and not others. Some authors like Quine have tried to make a distinction between 'real' mathematics and 'merely recreational' mathematics and we can imagine a similar distinction being made in this area. Perhaps this is defensible when combined with a holistic view that to have some of mathematics we need Set Theory to justify it and this gives us the rest of mathematics or some other similar argument. Perhaps this view is ultimately defensible, but if it is, the indispensability argument is being combined with some other strong views on coherence and ontology. If the argument for mathematical realism depends solely on physical objects instantiating certain relations, then the view is going to be markedly different from a standard realist view.

What if mathematical objects existed necessarily?

The major source of the problem with contingent abstract objects is that our scientific theories would be the same regardless of whether these abstract objects existed or not. However, you might deny that this troubling counterfactual really makes any sense at all. You might think, as I think most of us do, that mathematical objects exist necessarily and the indispensability argument is really about showing how it is that we can know that they exist.

The standard Platonist view holds that mathematical objects exist necessarily. Should a person holding such a view be persuaded by the indispensability argument? One question to ask is why believe that they do exist necessarily. If one has apriori arguments for mathematical realism, then this argument is adding nothing. However, one might believe only that if mathematical realism were true, then it must be necessarily true or conversely, that it might be necessarily false. This person can potentially be persuaded by the indispensabilist's claims.

While the view is consistent, there is a serious risk of the necessity undermining the indispensability thesis. If mathematical objects exist necessarily, it would seem that sentences about them must be necessarily true. But as Bob Hale points out in *Abstract Objects*, this implies that mathematics is conservative in the sense that Hartry Field uses. If Field is right, then the fact that it is conservative undermines its indispensability and thus the whole argument.

The concept of conservativeness is defined as follows: A mathematical theory M is conservative if, for any nominalistic assertion A, and body of such assertions N, A is not a consequence of N+M unless A is a consequence of N alone. Thus M doesn't 'add anything new' to N except mathematical statements and their mathematical consequences. It is clear why Field uses this notion – if mathematics is conservative, then it would be okay to use mathematical assertions in deducing nominalistic conclusions even if the mathematical lines were false since, in a certain sense, among nominalistic sentences, they are truth-preserving.

The fact that mathematics would be conservative follows directly from the fact that mathematical claims are necessary. Assume that mathematics is not conservative. Then there would be have to be some nominalistic sentence A such that N+M entailed A but N does not entail A. This would mean that N&~A is consistent. But M implies that N&~A is false. That means that M has contingent consequences. This would be impossible if M were composed only of necessary truths. So assuming mathematical truths are necessary, mathematics would be conservative.

Is Field right that if mathematics were conservative this would undermine the indispensability argument? One point is that if mathematics were conservative, then any conclusion that we come to about the physical world using mathematics was already strictly implied by just the non-mathematical premises that we used. So the situation is this: If it were the case that mathematical statements are necessarily true, then they cannot be indispensable to our best scientific theories in the sense that they are not doing any of the 'work' in entailing consequences. We may use mathematics to facilitate seeing these consequences, but strictly speaking, the theory is equally as powerful without it.

There is perhaps one important assumption missing from this claim. In order to even formulate the conservativeness claim as an objection against indispensability, it must be the case that there is some suitable body of nominalistic assertions that can be separated from the mathematical body of assertions. However, we cannot simply pull out all of the claims that

quantify over abstract objects since some of those may be contingent and important to the deductions. Perhaps there is an important scientific claim that says there is a function relating two physical groups of objects in a certain way. This sentence certainly appears to be contingent. Perhaps it can be used to prove certain things about one physical group given things about the other. This would be a nominalistic statement. But it seems as least conceptually possible that there may be no body of purely nominalistic statements that together imply that the groups have these properties that do not also imply that there is a function relating these two.
What I think could be done here is similar to the comments made about the antipodal weather points case. How could it be true that there is a function relating certain physical groups? It seems clear that the reason that there is such a function is that the groups each have certain physical properties that together entail that there would be such a function if there were functions at all. It is these physical properties that guarantee that the groups have certain other properties, not the fact that there is a function relating the two. Perhaps the function can help us to understand why these groups have certain physical properties, but the existence of such a function cannot entail something about the physical groups that was not already entailed by their physical properties alone. So the mixed ontological sentence can be rewritten into a nominalistic sentence and a mathematical sentence after all.

What can certainly be shown out of all of this is that if you hold the view that if mathematical objects existed they would exist necessarily and you hold the view that it is in principle possible to separate all of a theory's nominalistic content from its mathematical content, then you cannot hold that the indispensability argument has any sway. This is because the mathematical portion of your theory is a body of necessary truths and is thus conservative and thus in principle is dispensable from the theory.

Why is mathematics indispensable?

The obvious question to ask in all of this seems to be "Just why is positing mathematical objects indispensable to science?" Answering this question would go a long way toward answering our question of whether we are justified in thinking abstract objects exist because they are indispensable. One cannot simply point out that mathematics is applicable to the real world and is therefore true and then infer realism. What is needed is some explanation of how mathematics is applicable and why it would not be applicable if false. As stated before, indispensability should provide evidence for realism only if it is combined with a view that it would not be indispensable if false. It is also required that the sense in which the mathematical statements used have to be true cannot be captured by the fictionalist who might grant that 2+2=4 is true, because they believe that this statement is 'true in the story of mathematics.' What is required is that to be indispensable, the mathematical theory used would have to be true in the literal sense.

Even if mathematical theories were conservative it is undoubtedly the case that we would continue to use mathematics to justify our scientific conclusions. Why is that? It may be simply because we don't know how to do otherwise, but the best case for the indispensabilist would be that not only do we practically need it, but that there is actually no way to carry out the argumentation without it. Perhaps it is true that the nominalistically stated premises actually do guarantee the truth of whatever conclusion we would come up with, but that there is no way to carry out a deduction without using mathematics. It seems possible that this limit on deductions is not just a limit on the deductive power of human beings but that there is actually no way to do the deduction without referring to mathematical truths. Certainly when proving things in mathematics it is often the case that we must make reference to some logical principles to even get off the ground. There might not exist any possible proof without using them – even though the conclusion is strictly implied from the premises given. Thus the theory without mathematics would not be attractive since it doesn't have predictive power in the same sense that it does with mathematics and thus mathematics turns out to be indispensable after all. However, the fact that we need to use mathematics to deduce conclusions and make predictions gives us no argument that we could not do the very same thing if mathematics was simply a useful fiction and its statements were not actually true.

An alternative view is that mathematics is indispensable because we don't have any other way available to explain why things are the way that they are. Inference to the best explanation is often used to justify scientific realism, can it not be used to justify mathematical realism as well? In the case of physical theory, the existence of a physical object is part of the explanation of some event because when we attribute causal power to some entity in our explanation, it is required that this entity actually exists in order to have this power. On the other hand, we may say that 'the temperature was 0 degrees centigrade' as part of an explanation of why some water froze, but it is not clear at all that this explanation requires the positing of 0s existence. The view that for any mathematical explanation of a regularity in nature there was an equally good explanation without reference to the actual existence of mathematical objects defeats the indispensabilist here, thus the indispensabilist is forced to claim that there is no way to explain the facts without reference to mathematical ontology. This is the claim that Colyvan and others need to make and do explicitly make in order to defend their view. However, Colyvan's example of antipodal weather points is not a promising start.

Here are two examples that may undermine the notion of explanations giving us reason to believe mathematical realism. Now that we understand how to relate geometrical figures to algebraic structures many more explanations in mathematics are possible. For example, there are three famous problems that the Greeks never managed to solve with a compass and an unmarked straight edge: trisecting an angle, 'squaring the circle', and 'doubling the cube.' No explanation seemed possible as to why these tasks could not be carried out. Now that we know how to map geometrical objects onto a Cartesian coordinate structure, we can show that these tasks are in fact impossible and give an explanation why – doing so would require the ability to, in effect, solve cubic equations with these devices. But it can be shown that this cannot be done. Straight edges allow only linear solutions and compasses raise the possibilities to solving second degree equations. Cubic equations cannot be solved with these devices. Although it may be the case that it is actually impossible to explain why this cannot be done with reference to purely geometrical notions, it seems as though it is still a logical consequence of them with or without the algebra.

Does this explanation require that the algebraic objects actually exist? It seems to me clear that it doesn't. What about the following explanation: "Imagine that there were a structure like the real numbers. If there were such a structure, we could map our geometrical notions onto it and it would preserve all of the truths of geometry. If there were such a structure, it would show why these tasks were impossible, etc." Thus we can imagine constructing the reals as a device to help us better understand geometry. They need not exist independently in any real ontological sense. This explanation would certainly convince a contingent nominalist like Field who takes synthetic geometry to actually be a good model for how the nominalist program should be carried out.

Is there any reason to prefer the first explanation that actually posits the existing real number line? In a way it seems simpler, more elegant, etc. On the other hand, it is less parsimonious in a certain sense since the second explanation does not require the real numbers to actually exist. In any case, since both explanations are possible, it doesn't seem to me that we should prefer the first and by this preference commit ourselves to accepting the existence of the real numbers. We could just as easily posit the existence of the real algebraic numbers or some other structure isomorphic to the reals (or even a structure 'weakly isomorphic' where the new structure has all of the properties that matter to this particular problem) for this explanation to work. It seems clear to me that we don't actually need the real numbers to exist to explain this particular fact about geometry.

Here is another example that may seem more convincing to some, but is bound to be much more muddled for others. Causal explanations often use counterfactual information to help justify their causal claims. These explanations often involve reference to possible worlds. For example, we might say that the explanation of why A caused B involves the counterfactual 'If A had not occurred, then B would not have occurred.' In explaining why this counterfactual is true we might say that at the nearest possible world where A does not occur, B does not occur in that world either. Science also uses talk of possible worlds at least implicitly in some explanations if not explicitly in others.[3]

Does this require us to postulate the existence of possible worlds? Again, I think that the answer is no. Colyvan does refer to this slippery slope argument at the very end of his book. David Lewis uses it as an argument for modal realism. Some have used it as a reductio against the indispensability argument. What Colyvan argues, and needs to argue, is that although it may seem counterintuitive, if possible worlds talk was in fact indispensable to science, then we should believe possible worlds exist as well. The only debate in his mind is whether or not they truly are indispensable. It seems clear to me that even if there are no existing possible worlds other than the actual world, we might still use the talk as a useful theoretical tool in exactly the same way that we use the talk now. In fact, we may not be able to justify our conclusions in any other way. Since possible worlds are conceptually possible, it seems as though they may help in explaining phenomena even if they don't exist. In fact, it even seems possible to me that they are indispensable to our scientific theories in the sense that our best theories may make reference to them. Yet I think this is no reason to think that they really do exist since even if they didn't, talking about them would still have the same effect and the theories would have the same power as before. We have everything we need for the explanation containing in the conditional statement "if they did exist…".

It seems as though what the continuous functions, algebraic structures, and possible worlds are really doing in these cases is providing another way of looking at the problem to help us understand what consequences certain properties of the objects have. If this is all that mathematics ever does, provide a framework for understanding, it seems to me that there is no reason to think that using the framework for explanation is ontologically committing yourself to the view that there are actually existing abstract objects that can also be described by that same framework.

Who's left?

So if we hold to the traditional view that mathematical objects are abstract entailing at least that they are acausal, independently existing objects, then it seems as though there is no reason to believe that the indispensability argument gives us any justification for believing that such objects exist. If we believe that mathematical truths are necessary, then mathematics is a conservative theory, undermining its indispensability. If we believe that mathematical statements are true contingently, then although they may be indispensable to science, they would be equally as indispensable if they were false thus undermining the claim that indispensability is evidence for their existence.

Not only does the indispensabilist have to be a specific sort of strong scientific realist about the relationship of scientific theories to the world, but she is faced with a dilemma as well: In order to be persuaded by any form of empirical evidence for the existence of mathematical objects, she must believe either that they have causal powers (or some other forces-like property) in that the world would be different if they did not exist, or she must believe that they depend for their existence on objects in the physical world. Either view requires a drastic alteration in the way we ordinarily think of mathematical objects, alterations that few of us would be willing to make.

Notes:

1) For a much more full version of this argument, see Alan Baker's "The Indispensability Argument and Multiple Foundations for Mathematics."

2) In *Naming and Necessity*, Kripke argued that some identity statements are true necessarily such as water is H2O and Hesperus is Phosphorus but since these statements are empirical in nature, we can only know them aposteriori.

3) Colyvan points to the use of phase planes which can be best described as spaces of possible initial conditions (and thus arguably, implicitly invoking possible worlds) and the explicit invocation of possible worlds to account for certain problems in quantum mechanics concerning the collapse of the wave function and also in cosmology to account for the so-called fine-tuning problem. (Colyvan, 2001 pg. 155 who refers to Campbell, 1994, pp. 37-38.)

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